

Monitorability Bounds via Expander, Sparsifier and Random Walks

The Interplay Between On-Demand Monitoring and Anonymity

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Abstract

Software-defined networking (SDN), network functions virtualization (NFV) and network virtualization (NV) build a mini-cosmos inside data-centers, cloud providers and enterprises.

The network virtualization allows new on-demand management capabilities, in this work we demonstrate such a service, namely, on-demand efficient monitoring or anonymity. The proposed service is based on network virtualization of expanders or sparsifiers over the physical network. The defined virtual (or overlay) communication graphs coupled with a multi-hop extension of Valiant randomization based routing lets us monitor the entire traffic in the network, with a very few monitoring nodes.

In particular, we show that using overlay network with expansion properties and Valiant randomized load balancing it is enough to place $O(m)$ monitor nodes when the length of the overlay path (number of intermediate nodes chosen by Valiant's routing procedure) is $O(n/m)$. We propose two randomized routing methods to implement policies for sending messages, and we show that they facilitate efficient monitoring of the entire traffic, while avoiding traffic bottlenecks. We show that the traffic is distributed uniformly in the network and each monitor has equiprobable view of the network flow such that there is no preferable vertex for monitor location. In terms of complex networks, our result can be interpreted as a way to enforce the same betweenness centrality to all nodes in the network.

Beyond on-demand monitoring, we have identified that our results are useful in serving the somewhat complement application, namely anonymity. Thus, we propose monitoring or anonymity services, which can be deployed and shut down on-demand. Our work is the first, as far as we know, to bring such on-demand infrastructure structuring using the cloud NV capability to existing monitoring or anonymity networks. Incorporating recent technological advances in networking in order to provide dynamic setup and reconfigurations of network topology for monitoring or anonymity services. We propose methods to theoretically improve services provided by existing anonymity networks, and optimize the degree of anonymity, in addition providing robustness and reliability to system usage and security.

At last, we believe that, our constructions of overlay expanders and sparsifiers weighted network, that use several random walk trees are of independent interest.

Index terms— *sdn, nf, naas, on-demand, monitoring, anonymity, network, expander*

1 Introduction

Software-defined networking (SDN) is a building block and mechanism for realizing network virtualization (NV) and architecture independent network functions virtualization (NFV). Network virtualization allows us to create virtual networks which network's topology is decoupled from the topology of the underlying physical network, and dynamically create policy-based virtual networks [39]. Overlay network is one of possible forms of network virtualization. Its main idea is to encapsulate a network service decoupled from the underlying infrastructure [39].

Network functions virtualization concerns implementation of network functions in software. Due to increasing demand for dynamic network architectures, network virtualization technology was utilized in Network-as-a-Service (NaaS) models that enabled dynamic deployment of a network service on-demand [7]. Integration of SDN and NFV enables flexible, programmable and dynamic deployment of network services [6], and in [7] one such SDN based implementation was presented.

Today, companies heavily rely on networking and Internet connection even for simple tasks. As such, traffic monitoring is a common request in private corporate networks. In this work, we describe an on-demand overlay network construction which enforces a defined policy of message sending scheme utilizing Valiant [46] randomization technique. We propose to construct an overlay network over the underlying network such that the overlay network will preserve the expansion properties of the underlying network, moreover we build the overlay network oriented towards having capacity approximately close to the capacity of the underlying physical network. This requirement makes the overlay network maximally utilize the capacity usage for all sort of network applications.

The monitoring of the network is resorted to monitors that are located at the nodes of the constructed overlay network. Monitors are not required to be located at each node of the network, but can be placed at selected nodes. This decision depends on selected message sending policy of the network. The combination of the enforced message sending scheme and spectral properties of the overlay network graph provides uniform monitoring abilities such that each monitor supervises uniform fraction of network traffic, and none of the monitors have higher network traffic observation.

Goyal et al. showed in [30] that it is possible to construct an expander graph via random spanning trees. We follow this method of constructing expander graph from random spanning trees. However, we employ this method on weighted graphs in order to build a capacity biased overlay network. Moreover, we show that the constructed graph is a sparsifier of the underlying graph, namely, the constructed graph spectrally approximates the underlying graph. Further, we provide a distributed algorithm for overlay network construction together with an algorithm for verifying the mixing properties of the constructed graph. In contrast to independent sampling methods of sparse graph construction, spanning tree is a connected graph, and as such, using spanning trees results in a connected graph.

By its very nature, the constructed network is robust, and is able to recover from congestion and link failures due to expansion properties, and due to path diversity which is achievable by combination of random spanning trees [30, 36]. Moreover, the overlay network is scalable and may grow linearly in the number of vertices.

We have also identified a possible application of our dynamic network architecture service for anonymity in the network. The proposed network architecture with its spectral properties and enforced messaging policy provides high degree of anonymity for senders as for receivers. Its on-demand flexibility allows private deployment and in seldom use or with trusted party.

In the next session we discuss the related works. Later, we describe the overlay network construction method and distributed expansion verification algorithm used in distributed construction of the overlay network. In Sections 5 and 6 we describe the use of expander graphs for monitoring, message sending scheme and the probabilities of successful network monitoring. Finally, in Section 7 we show a feasible implementation of our on-demand network construction as an on-demand anonymity service, providing innovation to anonymity networks research area.

2 Related Work

Goyal et al. demonstrated the building process of an expander graph, which expansion depends on the degree of the primal graph, by union of random spanning trees and showed that the union of the trees approximates each cut of the primal graph within a factor of $O(\log n)$. In our

work, we show that the same method can be used to build weighted expanders from a weighted graph, moreover, we show that the union of weighted spanning trees spectrally approximates the primal graph.

Similar result to ours was shown by Fung and Harvey in [25]. Fung et al. showed that sampling spanning trees, while adjusting their link weights, results in a sparsifier. In contrast to their work, we utilize the weights of the given primal graph in order to construct a capacity optimized expander, while including enough edges results in an overlay network which sparsifies the primal graph. Performing random walk on a weighted graph, better preserves the locality¹ of a cut.

In contrast to previously mentioned works, we additionally employ a distributed construction of the expander graph. Our construction method can run in a distributed manner for concurrent construction of expander graphs. Distributed construction of expander graphs was also shown by Dolev and Tzachar in [19], where the authors introduced the notion of *Spanders*, distributed spanning expanders, and showed a practical way for verifying that the constructed graph is an expander. We use the method of expansion verification presented in [19] in order to optimize the construction of the overlay network and limit the number of constructed spanning trees.

We show that our network architecture is valuable for network monitoring. Altshuler et al. showed an efficient flooding scheme for generating a collaboration between a group of random walking agents who are released from different sources and at different times [3]. This participation of agents results in a collaborative monitoring infrastructure, requiring only a small number of active monitors.

Measures for estimating monitoring capabilities of a vertex (BC, SPBC, LC, FBC) were established in [22, 26, 5, 23]. Routing betweenness centrality (RBC), a network measure for estimating the control probabilities of a vertex or a set of vertices was proposed in [17], generalizes aforementioned network measures. RBC measures the extent to which vertices or group of vertices are exposed to the traffic [17]. As a result, RBC is useful for predicting the effectiveness (and cost) of passive network monitoring. We use the RBC measure to show that each monitor in our constructed network is equivalently effective as a passive monitor.

Our results can be used as the base to provide a flexible and robust network architecture as a service with on demand deployment. Boubendir describes an implementation of NaaS architecture with SDN-enabled NFV in [6], and shows feasible on demand dynamic network service based on SDN-enabled NFV [7]. We further exploit the NV, SDN and NFV emerging technologies to enable network architecture as a service for use in private commercial networks, network and service providers, or facilities desiring flexible policy enabled networking for securing their traffic and monitor network flows for mitigation of misuse or malicious uses.

3 Preliminaries

3.1 Notation

We specify a graph $G = (V, E)$, having vertex set $V = 1, \dots, n$ and edge set $E \subseteq \{(u, v) \mid u, v \in V\}$. If the graph is weighted, it will be specified by $G = (V, E, w)$ where $w_{(u,v)} > 0$ for each $(u, v) \in E$. For each vertex $u \in V$, $w(u) = \sum_z w(u, z)$ is defined as the total weight of edges that incident to the vertex u . We also define the total weight of the graph as the sum of all weights

$$W = \sum_{e \in E} w(e)$$

We may denote by N the set of all nodes in a network and C as the set of non-monitoring/compromised nodes.

¹We thank Nogal Alon for observing this.

For $S, T \subset V$, we specify the set of edges emerging from S to T by

$$E(S, T) = \{(u, v) \mid u \in S, v \in T, (u, v) \in E\}$$

We denote the *edge boundary* of a set S as ∂S and is defined as $\partial S = E(S, \bar{S})$. The edge boundary is a set of edges emerging from the set S to its complement. We specify the set of neighbors of v for $v \in V$, as $\Gamma(v) = \{u \in V \mid (u, v) \in E\}$. For $A \subseteq V$, $\Gamma(A) = \cup_{v \in A} \Gamma(v)$ and $\Gamma'(A) = \Gamma(A) \setminus A$.

3.2 Graph Expansion

For a good introduction on expander graphs see [32, 27]. Here, we describe the only basic definitions of expander graphs. Expansion requires that any set of vertices, of size at most $n/2$, has a relatively large set of neighbors. The edge expansion ratio of G , denoted by $h(G)$, is defined as:

$$h(G) = \min_{\{S \mid |S| \leq \frac{n}{2}\}} \frac{|\partial S|}{|S|}$$

The vertex expansion of graph G is defined as:

$$h(G) = \min_{\{S \mid |S| \leq \frac{n}{2}\}} \frac{|\Gamma'(S)|}{|S|}$$

4 Expander Overlay Network Construction

Assume that the underlying network is represented by a weighted graph $G = (V, E, \omega)$. A random walk begins from a randomly chosen vertex, and moves to one of its neighbors with probability proportional to the weight of the edge, $P[(u, v)] = \frac{\omega(u, v)}{\omega(u)}$. Each time the random walk arrives to a new vertex, a vertex which was not visited before; the edge, through which it arrived, is added to the spanning tree construction. Inspired by the work of Goyal et al., we show that for a weighted bounded degree graph and for the weighted complete graph a small number of spanning trees result in a subgraph with expansion properties comparable to each cut of the original graph. In addition, the generated subgraph utilizes the capacity of the primal network by including edges of greater capacity with a greater probability.

For generating a random spanning tree, we use the algorithm derived by Andrei Broder [8]. However, we modify the algorithm to utilize the probabilities of weights (Algorithm 1). Overlay network is created by repeating k times the creation of a random spanning tree (Algorithm 2).

Algorithm 1: Generation of Spanning Tree via Random Walk Simulation

input : weighted graph $G = (V, E, \omega)$

output : collection of edges T

- 1 $T = \{\}$
 - 2 Simulate a weighted random walk on graph G starting at an arbitrary vertex s until every vertex is visited.
 - 3 $T \leftarrow T \cup e(v, u)$ for each vertex $u \in V \setminus s$ if this is the first visit of vertex u .
 - 4 **return** T
-

Goyal et al. showed in [30] that for any bounded degree graph, the union of at least two random spanning trees of the graph approximates the expansion of every cut in the graph. Using more trees gives a better approximation. We follow the outline of the proofs in [30] and prove them for weighted graphs. Pemantle proved the following theorem on the negative correlation property of uniform spanning trees [43].

Algorithm 2: Expander Overlay Construction

input : weighted graph $G = (V, E, \omega)$, k
output : union of k random spanning trees U_G^k

- 1 $U_G^k = \{\}$
- 2 **repeat**
- 3 Generate a random spanning tree T , according to Algorithm 1
- 4 $U_G^k \leftarrow U_G^k \cup T$
- 5 **until** k trees are generated
- 6 **return** U_G^k

Theorem 4.1. *For any finite connected graph G , let T be a uniform spanning tree. If e and f are distinct edges, then*

$$P[e, f \in T] \leq P[e \in T]P[f \in T]$$

Goyal et al. extended Theorem 4.1 for any subset of edges in [30]:

Negative Correlation of Edges's *For any subset of edges $e_1, \dots, e_k \in E$ we have*

$$P[(e_1 \in T), \dots, (e_k \in T)] \leq P[e_1 \in T] \cdots P[e_k \in T] \quad (1)$$

We prove our results using Chernoff bounds, for this, we define the following indicator variables:

$$X_e = \begin{cases} 1 & e \in T \\ 0 & \text{otherwise} \end{cases}$$

Now, we can rewrite (1) as

$$E[X_{e_1} \cdots X_{e_k}] \leq E[X_{e_1}] \cdots E[X_{e_k}] \quad (2)$$

We can say that $\{X_e\}$ satisfying (2) are negatively correlated for every subset of edges e_1, \dots, e_k . It is possible to apply Chernoff bounds unaltered to negatively correlated variables [20, Proposition 5]. Then, by [2, Theorem A.1.13] it is possible to derive the following version of Chernoff bounds [30]:

Theorem 4.2. *Let $X_{i=1}^n$ be a family of 0 – 1 negatively correlated random variables such that $1 - X_{i=1}^n$ are also negatively correlated. Let p_i be the probability that $X_i = 1$. Let $p = \frac{1}{n} \sum_{i \in [n]} p_i$. Then for $\lambda > 0$*

$$P\left[\sum_{i \in [n]} X_i < pn - \lambda\right] \leq e^{\frac{-\lambda^2}{2pn}}$$

Base graph is a complete graph. Next, we show that the union of two random spanning trees is required to approximate the expansion of the complete weighted graph.

Theorem 4.3. *The union of two randomly spanning trees of the complete weighted graph on n vertices has constant vertex expansion with probability $o(1)$.*

Proof. In general, if all weights are divided by the same constant, none of the probability measures change, therefore the case of rational weights can still be thought of as corresponding to a uniform random spanning tree [34]. Starting from a random vertex and moving to a random neighbor, we can inspect a random walk as a random selection of one neighbor for each $v \in V$. However, the choice of vertices is not completely independent. Following, all edges of the cut are inspected as a random negatively correlated selection of edges. The proof is analogous to the proof in [30]. □

Base graph is a bounded-degree graph. We consider weighted and irregular graphs with bounded degrees, and we show the probability bounds for the construction process on these graphs.

Theorem 4.4. *For a weighted graph $G = (V, E, \omega)$, let U_G^k be the union of k uniformly random spanning trees. Also, let $\alpha > 0$ be a constant and $\alpha(k-1) > \frac{8}{p}$. Then with constant probability, for every $A \subset V$ we have*

$$|\partial_{U_G^k} A| \geq \frac{1}{\alpha \ln(n)} |\partial_G A|$$

Proof. We follow the proof in [30] while modifying the parts referring to the weighted characteristics of the graph. We refer to random variable T_G as the spanning tree constructed on the graph G . If we start the random walk at vertex u , then with probability $\frac{w(u,v)}{w(u)}$ the first traversed edge is (u, v) which then get included in T_G . In order to calculate the expectation of $|\partial_{T_G}(A)|$, we need the probability for including an edge in the spanning tree, $P[e \in T_G]$. We use the *Matrix Tree Theorem* [43] in order to find $P[e \in T_G]$

Theorem 4.5 (Matrix-Tree Theorem). *Denote by $\kappa(G)$ the sum of the weights of all spanning trees of G . Let G be a finite simple, connected, weighted graph without loops, with Laplacian matrix $L = L(G)$. Let L_x denote L with the x_{th} row and column removed. Then*

$$\det L_x = \kappa(G)$$

Let $G_e = G \setminus \{e\}$ be the obtained graph after deletion of the edge e . The probability for an edge e being part of a random spanning tree will be

$$P[e \in T_G] = 1 - \frac{\det L_x(G_e)}{\det L_x(G)}$$

and the average probability is

$$p = \frac{\sum_{e \in E} P[e \in T_G]}{|E|}$$

Therefore, for $A \subset V$

$$E[|\partial_{T_G} A|] = p |\partial_G A|$$

We have $|\partial_T A| = \sum_{e \in \partial_G A} X_e$. The random variables X_e are negatively correlated, and this allows us to use Theorem 4.2:

$$P\left[\sum_{e \in \partial_G A} X_e < p |\partial_G A| - \lambda\right] < e^{-\lambda^2 / (2p |\partial_G A|)}$$

Where the average of $P[X_e = 1]$ is p for $e \in \partial_G A$. For $\lambda = \frac{1}{2} p |\partial_G A|$ we have

$$P\left[|\partial_{T_G} A| < \frac{1}{2} p |\partial_G A|\right] < e^{-\frac{1}{8} p |\partial_G A|}$$

which gives the following bound for k trees

$$P\left[\partial_{U_G^k} A < \frac{1}{2} p |\partial_G A|\right] < e^{-\frac{k}{8} p |\partial_G A|} \quad (3)$$

The expected number of sampled edges in a cut must be at least $\ln(n)$ [33]. We would like to estimate the probability that there is a bad cut. For the cut to be bad, we need to look at all cuts of size a in the first random tree, while these cuts have size of at least $\alpha a \ln(n)$ in graph G . In other words, assume that there's a cut A such that $|\partial_{U_G^k} A| = a$ and $|\partial_G A| \geq \alpha a \ln(n)$. These cuts have to be small in all remaining trees. The probability for this is given by (3). The

number of cuts of size a in the first tree is at most $\binom{n-1}{a}$. The probability that a bad cut A exists

$$\begin{aligned}
& \sum_{a=1}^{n/\ln(n)} \binom{n-1}{a} e^{-(k-1)\alpha a \ln(n) \frac{1}{8}p} \\
& < \sum_{a=1}^{n/\ln(n)} \binom{n}{a} e^{-(k-1)\alpha a \ln(n) \frac{1}{8}p} \\
& \leq \sum_{a=1}^{n/\ln(n)} \left(\frac{ne}{a}\right)^a e^{-(k-1)\alpha a \ln(n) \frac{p}{8}} \\
& = \sum_{a=1}^{n/\ln(n)} \exp\left(\ln\left(\frac{ne}{a}\right)^a + \ln(e^{-(k-1)\alpha a \ln(n) \frac{p}{8}})\right) \\
& = \sum_{a=1}^{n/\ln(n)} \exp\left(a\left(\ln\left(\frac{e}{a}\right) + \ln(n)(1 - (k-1)\alpha \frac{p}{8})\right)\right)
\end{aligned}$$

Choosing $(k-1)\alpha > \frac{8}{p}$ makes the above sum $o(1)$. □

U_G^k is a sparsifier. We show that U_G^k generated by the process defined in Section 4 is a sparsifier of a weighted graph G , if $|U_G^k|$ is sufficiently large.

Theorem 4.6. *Let G have Laplacian L and U_G^k have Laplacian L' and $\frac{1}{\sqrt{n}} \leq \epsilon \leq 1$, $|U_G^k| \in O(n \log n / \epsilon^2)$. With constant probability*

$$\forall x \in \mathcal{R}^n \quad (1 - \epsilon)x^T Lx \leq x^T L'x \leq (1 + \epsilon)x^T Lx$$

Proof. Define the conductance function $\mathcal{C} : E \rightarrow \mathcal{R}^+$, \mathcal{C} : the weight of an edge e is taken to be its conductance $\mathcal{C}(e)$. The effective resistance R_e between the endpoints of an edge e , proportional to the commute time between the end-vertices of e and is equal to the probability that e belongs to a spanning tree [44].

During our process of U_G^k generation, if an edge e was traversed by the random walk and $e \in T$, it is equivalent to the probability of sampling the edge e according to its conductance and effective resistance, i.e. $p_e = \mathcal{C}(e)R_e$. It was proven by Spielman and Srivastava in [44] that sampling edges with probability proportional to $\mathcal{C}(e)R_e$, and adding scaled sampled edges to the output graph, results in a sparsifier output graph. Therefore, let L be a Laplacian of a random graph G and L' a Laplacian of U_G^k . Since $|U_G^k|$ is the number of times edges were independently sampled, if $|U_G^k|$ is sufficiently large, in particular if $|U_G^k| \in O(n \log n / \epsilon^2)$, then with a constant probability the quadratic forms of L and L' are ϵ -close. Since, each spanning tree is of size $O(n)$, it means that we need $O(\log n)$ random spanning trees to construct a sparsifier. □

4.1 Distributed Construction of Overlay Network

Algorithm 2 can be modified to a distributed algorithm for the construction of the overlay network. This is also beneficial for networks where a centralized entity has several cores that can execute the algorithm in a distributed manner. For example, in software-defined networking, the network control software [24] can execute the iterations of the algorithm concurrently or delegate the execution of some iteration to a different controller. Each controller will execute one or a few iterations of the algorithm until the required approximation of the primal graph is achieved. In such setup, a monitoring algorithm for verification of expander construction is

required. The monitoring algorithm can be executed by the main controller in charge of the network (see Alg. 4).

We exploit the mixing rate based monitoring algorithm described in [19] which is modified for our construction scheme (see Alg. 3). This monitoring algorithm is used to estimate the expansion of the constructed graph, since calculating the expansion is NP hard. The mixing rate based monitoring algorithm, employs the rapidly mixing property of expander graphs, $O(\log n)$ mixing rate. Following, the cover time of such graphs is $O(n \log n)$ [19]. The control software starts a random walk of length $O(n \log n)$ on an arbitrary vertex v , each new visited vertex is marked and counted. The neighbors of the newly visited vertex are added to the set of all neighbors. Afterwards, the walk proceeds from one of the randomly chosen neighbors. When the random walk terminates, the counter is examined by the control software. In case the walk covered less than n nodes, then we can conclude with high probability that the graph is not rapidly mixing as was required or there are too many edges in the constructed graph, implying that the construction was not successful [19].

Algorithm 3: Mixing Rate Based Monitoring

input : U_G^k for some k
output : j : number of counted nodes

- 1 L : max length of the walk
- 2 $counter \leftarrow 0$
- 3 v : arbitrary chosen vertex
- 4 $length \leftarrow 1$
- 5 $\Gamma'_{U_G^k} = \{\}$
- 6 **repeat**
- 7 **if** v was not visited before **then**
- 8 $counter \leftarrow counter + 1$
- 9 $\Gamma'_{U_G^k} \leftarrow \Gamma'_{U_G^k} \cup \Gamma'(v)$
- 10 **end**
- 11 $length \leftarrow length + 1$
- 12 choose $u \in \Gamma'_{U_G^k}$
- 13 $v \leftarrow u$
- 14 **until** all vertices are visited or $length > L$
- 15 **return** $counter$

Algorithm 4: Distributed Expander Overlay Construction

input : weighted graph $G = (V, E, \omega)$
output : union of k random spanning trees U_G^k

- 1 $U_G^k = \{\}$
- 2 $k \leftarrow 1$
- 3 **while** mixing rate requirement not satisfied **do**
- 4 $\mathcal{T} \leftarrow$ delegate the generation of k random spanning trees
- 5 **for** T in \mathcal{T} **do**
- 6 $U_G^k \leftarrow U_G^k \cup T$
- 7 **end**
- 8 verify mixing rate requirement using Algorithm 3
- 9 $k \leftarrow 2 \cdot k$
- 10 **end**
- 11 **return** U_G^k

Algorithm 3 can be sped up by performing $O(n)$ random walks of length $O(\log n)$ [19, 1]. The controller software, which runs the algorithm, in SDN can perform parallel random walks

and possibly delegate to other controller units. Each of the controller units will return the result to the main controller, upon which it will be decided to stop generating random spanning trees (see Alg. 4).

Algorithm 4 is executed until the expansion of U_G^k , measured by the mixing rate quality, is of the required value or at most $O(\log n)$ iterations. At each iteration of the algorithm, the number of generated spanning trees is doubled. The algorithm will stop if the required approximation of the expansion is achieved, which results in less spanning trees and less edges in the expander graph.

5 Sparse Expander Graphs for Monitoring

We propose a message sending scheme similar to the one proposed by Valiant in [46]. Source node s that sends a message to destination t , performs a random walk of length $l = \log(n)$ to intermediate destination $v \in V$. The set of vertices visited by a length l random walk on an expander graph is a randomly chosen sequence of v_0, v_1, \dots, v_l , where each v_{i+1} is chosen uniformly at random and independently, among the neighbors of v_i , for $i = 0, \dots, l - 1$ [11]. The sender sends the message along the path from s to v , such that the path is a sequence of vertices chosen by the random walk, and then the random walk is performed again to deliver the message from v to destination t . The message is sent through two paths, each of length $\log(n)$.

In fact, the sender can arbitrary choose the number of several intermediate destinations, such that a message would be sent from source to intermediate destinations v_1, v_2, \dots, v_r for some chosen r . Each path from source to intermediate destination and between the intermediate destinations, including destination t chosen by s , is a random walk of length $\log(n)$. Overall route complexity is $O(\log n)$ overlay edges. If the underlying graph is an expander graph too, then the total route complexity is $O(\log n)$, otherwise it is $O(\log(n) + \text{diam}(G))$.

The message sending scheme implements two routing methods; incremental path building (incremental routing) and loose routing. In incremental routing, the sender's application proxy software builds the route hop by hop, corresponding and exchanging keys with each router on behalf of the user. As such, sender's application proxy is aware of each participant in the chosen route. In loose routing [29], a node may make a decision to extend or change the message's path, this means that the sender may not be aware of all the nodes which constitute the whole path.

On the one hand, it may be preferable for the sender to route a message to a subarea of the network where the local routing policy will decide upon the routing in the subarea towards the receiver. On the other hand, in the former case the implementation is easier. Further, the sender can negotiate session keys with each router it chose in the route, as opposed to the latter case where it is left to the router which performs modification to the route on behalf of the sender.

Incremental path building. Suppose, there are C out of N nodes which are not monitor nodes. We show that, one monitor located on a path of length $\log(N)$ is enough to monitor the network flow. We estimate the probability for at least one node along the whole path is a monitor node. The probability for the overall route to be not monitored, i.e. composed of oblivious nodes only is given by

$$P[\text{oblivious}] = \prod_{i=0}^{r \cdot \log(N)} \left(\frac{C-i}{N-i} \right)$$

Therefore, an overall route will have at least one monitor node with probability

$$P[\overline{\text{oblivious}}] = 1 - P[\text{oblivious}] = 1 - \prod_{i=0}^{r \cdot \log(N)} \left(\frac{C-i}{N-i} \right)$$

Using the message sending scheme from Section 5, it is highly unlikely for the whole route to be unmonitored. For example, let $G = (|V| = 1021, E, d = 3)$ expander graph where $C = 700$ nodes are non-monitoring nodes. The sender sends a message with two intermediate destinations, $r = 2$. Each random walk would be of length $\lfloor \log_d(N) \rfloor = 6$ and the overall route would be $3 \cdot \lfloor \log_d(N) \rfloor = 18$.

$$P[\overline{oblivious}] = 1 - \prod_{i=0}^{18} \left(\frac{700 - i}{1021 - i} \right) = 0.99$$

There is 99% chance for at least one node to be a monitor node.

Loose routing. Suppose, $G = (V, E)$ where the vertices in some subset $C \subseteq V$, $|C| = \beta N$, are not monitored. We would like to examine the probability that the random walk will be composed of non-monitoring nodes only. Assume a network flow begins at node s_0 and advances using a random walk on v_1, v_2, \dots, v_t . Denote by (C, t) the event that this random walk is confined to C , i.e. $\forall i \ v_i \in C$. The probability of the event (C, t) is given by [32]

$$Pr[(C, t)] \leq \beta^t$$

There, it is exponentially unlikely that only non-monitoring nodes were traversed by the random walk. Hence, with high probability, at least one node will be a monitor. We can bound the bad set C , assuming that we want the event (C, t) be at most 0.5 for given t :

$$\left(\frac{C}{N}\right)^t \leq 0.5 \rightarrow C \leq \sqrt[t]{0.5} \cdot N$$

For example, for $N = 1021$ and $t = \lfloor \log(N = 1021, d = 3) \rfloor = 6$ we can have up to $C \leq 910$ non-monitoring nodes.

The probability calculations above, show that with high probability in a route of length $O(\log N)$ at least one node is a monitoring node. Consequently, $O(N/\log N)$ monitors are required in order to monitor the traffic in the network with high probability.

The above calculations are slightly relaxed, we can improve their accuracy using Chernoff bounds which can be applied unaltered to hyper-geometrically distributed binary variables [16]. If the random walk in the message sending scheme is allowed to visit each vertex more than once, then the following calculation are either in hold, since in that case the binary variables would be independent. In particular, we inspect a random walk of length t vertices from the set N , and C the set of non-monitoring nodes. We can define an indicator variable, X_v , which equals 1 if a node is a monitoring node and otherwise 0; For $v \in V$ define

$$X_v = \begin{cases} 1 & v \notin C \\ 0 & \text{otherwise} \end{cases}$$

Let p_i be the probability that $X_i = 1$, and r will be the number of intermediate nodes in the message sending scheme. Let $X = \sum_{i=1}^t X_i$, where $t = r \cdot \log N$ is a total number of selected vertices during the random walk. It is easy to see that $\mu = E(X) = t \cdot \frac{N-C}{N}$. Let $0 < \delta < t - \mu$, then

$$Pr[X \leq \mu - \delta] \leq e^{-\frac{2\delta^2}{t}}$$

the above shows that the probability for a path to be completely not monitored is exponentially small.

6 Measuring Monitoring Success

In this section, we quantify the monitoring level of the system. We show that our system is able to uniformly monitor the traffic in the network. One approach, which we use, is based on

Bayesian inference of traffic analysis, where we try to estimate the probability of communication flow path in the system by sampling the hidden state of the system. In the second approach, we use information theory metrics, similar to methods proposed in [13] and independently in [42] for measuring anonymity of a network. We estimate the degree of monitoring/anonymity by assessing the distribution of the auditing nodes.

Traffic analysis. In our system model, each of the message sending schemes has an analogous effect of a mix [10]. We want each monitoring node to audit as much traffic as possible. We would like to show that the sampling of the traffic in our system model is uniform and results in optimized network for monitoring. The formal definitions and notations of the system model which we follow, can be found in [45]. Assume the system model includes N_{mntr} monitors and there are N_{msg} messages in the system. A user selects a number of intermediate nodes according to the message sending scheme. These are the combined constraints, \mathcal{C} , of the system. The system is observed during the period T and N_{msg} messages are sent through the system and monitored by a monitor node. The monitor observing the system generates an observation \mathcal{O} . The goal is to determine the distribution of the messages in the system. The hidden state of the system is described by the path each message has taken. Therefore, sampling hidden states of the system is equivalent to sampling paths.

The user is allowed to choose the number of intermediate nodes, r . The number of intermediate nodes affect the total path length, L_x , of the message, and this length is a multiple of $\log(N)$. The user selects uniformly at random amongst the possible values r . Thus, the probability for selecting a path P_x given constraints \mathcal{C} is

$$\begin{aligned} Pr[P_x|\mathcal{C}] &= Pr[L_x = l|\mathcal{C}] \cdot Pr[M_x|L_x = l, \mathcal{C}] = \\ &= \frac{1}{r} \cdot \frac{r \cdot \log(N)}{N_{mix}} = \frac{\log(N)}{N_{mix}} \end{aligned}$$

and the probability of a hidden state would be

$$Pr[\mathcal{HS}|\mathcal{O}, \mathcal{C}] = \prod_{x=1}^{N_{msg}} Pr[P_x|\mathcal{C}] = \prod_{x=1}^{N_{msg}} \frac{\log(N)}{N_{mix}}$$

As one can see, the probability for every path of a message is equally likely. Consequently, the probability for each hidden state of the system would be equally likely too. The obtained samples of the hidden state can be used to compute the probability for monitoring network flows. Since all states are equiprobable, the distribution of the hidden states would be uniform and as such the distribution of the network flows.

Hence, each monitor can uniformly audit the network traffic, and the network traffic is unbiased towards flowing through specific links.

Monitoring avoidance. In this section, we would like to estimate the degree of monitoring on the expander based overlay network with our proposed message sending scheme. Following the precise definition based on information theory given by [13], the degree of monitoring is based on probabilities given by the sending nodes after observing the system and inferring a distribution which gives probabilities for auditing nodes.

According to the given definition, the maximum degree of monitoring is achieved when the sender sees all monitors as equally likely being the auditors of a message. The entropy of the system, after observation was performed, is compared against the maximum entropy. This comparison gives a hint of the amount of information that was learned.

The most useful property of expander graphs is that random walks mix fast. If a random walk starts at any vertex, after $O(\log N)$ steps the position will be uniformly random. Therefore, without any additional knowledge of the system, and given that the probability for each hidden state of the system is equally likely, each of the monitors is assigned the probability of $p_i = \frac{1}{N_{mntr}}$ being the auditor of the message. Further, without any prior knowledge on number of monitors,

every node in the graph may be a possible monitoring node. Hence, the probability of identifying the monitor node is $\frac{1}{N}$. The entropy of the system is maximized.

Monitors locations. We estimate the potential of monitoring for each node in the network, employing routing betweenness-centrality (RBC) measure proposed by Dolev et al. RBC of a vertex represents its potential to monitor and control data flow in the network. Following the definitions in [17], let $R(s, u, v, t)$ be the probability that u will forward to v a packet with source address s and target address t . All routing decisions in our message sending policy are independent. Let $\delta_{s,t}(v)$ be the probability that this packet will pass through the vertex v . $\delta_{s,t}(v)$ can be recursively computed for arbitrary $v \in V$:

$$\begin{aligned}\delta_{s,t}(s) &= 1 \\ \delta_{s,t}(v) &= \sum_{u \in \text{Pred}_{s,t}(v)} \delta_{s,t}(u) \cdot R(s, u, v, t)\end{aligned}$$

where $\text{Pred}_{s,t}(v)$ is a set of all immediate predecessors of v . Since the probability for a message to be forwarded from any vertex u to its neighbor v is equal for every pair of vertices and their neighbors, we would have equal values of $R(s, u, v, t)$ for each u and v and consequently the computed $\delta_{s,t}(v)$ would have a value equal for each node. Thus, we obtain the result that the RBC is equivalent for every node in our constructed network. Consequently, there is no preferred location for a monitor, and each node has equivalent potential for monitoring the network.

7 Anonymity

Cryptographic primitives are used to establish a secure communication channel and ensure confidentiality, provide digital signatures, message authentication codes, and hash functions. However, cryptographic primitives cannot provide anonymous communication since the channel is susceptible to traffic analysis. Encrypted messages can be tracked and reveal the two communicating parties. The information with regard to the identity of the communicating parties can be sensitive. For example, inter-company conversation, or email users may not wish to reveal who they are communicating with. In some cases, anonymity may be a desirable preference.

Extensive amount of work was done on the subject of anonymity networks and anonymity protocols. Chaum introduced the concept of a *mix*, which aim is to hide the identity of the sender [10]. The work by Chaum is fundamental to mix-net based protocols which use a set of mix servers, in particular, *onion routing* which was developed following the principles of mix cascades, and its second generation Tor [29, 28, 40, 15].

Chaum also proposed DC-net [9], a broadcast network which provides both sender and receiver anonymity. DC-net scheme suffers from poor scalability and is unsuitable for large-scale networks [31]. Most notable work based on DC-net is Xor-Trees [18], which was proposed by Dolev and Ostrovsky to provide sender and receiver anonymity, additionally reducing the amount of communication overhead.

Additional anonymous communication scheme which provides high degree of anonymity is Buses [4], which is a network routing based anonymous communication scheme, that can be viewed as a bus system. Buses attempts to hide traffic patterns and to provide an unlinkability for two communicating parties.

Recent work by Hermoni et al. proposed a Peer-to-Peer file sharing system which provides anonymity to all participants, namely, receiver (server) and sender (publisher or reader) anonymity [31]. Hermoni et al. proposes the use of anonymity tunnels for each different user. The authors assume a semi-honest adversary in their first provided solution, while the same adversary as the one assumed in Tor for their second solution.

In [14], the authors compare different network topologies to assess the impact of network topology on the anonymity of the network and the influence of the topology on the overhead

of link padding. The authors in [14], found that restricted topologies provide better anonymity with less cover traffic overhead (as oppose to unrestricted topologies such as a complete graph).

Additionally, Danezis in [12] proves that restricted network topologies scale better as the number of mix nodes grows. In order to provide the maximal anonymity, the required cover traffic grows linearly with the number of mix nodes. In addition, Danezis proposed to use sparse, connected graphs with constant degrees, based on expanders as a topology for anonymity networks.

Legal considerations play important role in deploying and using anonymity network. Governments practice classifying Tor users and traffic analysis on Tor traffic [41]. There are also countries that suppress Tor and other anonymity servers. Further, nodes locations are publicly known, and government agencies collect traffic from these nodes, in addition to utilizing de-anonymization techniques [35]. Anonymity network deployed by users with similar network usage and content demands (e.g., friends, family) is likely to have a higher proportion of usable servers, and less intimidated by legal requests to hand over server logs [37].

We show that sparse overall network with expansion properties can face the aforementioned challenges of anonymity networks, and provide anonymity service with optimal and restricted routes optimal topology.

Anonymity model and metrics. In the model proposed by [13], the concept of entropy was proposed to measure the information gained after an attack. Anonymity of a subject is defined as being not identifiable within the anonymity set [38]. The anonymity set is the set of all possible subjects who might cause an action. A subject is identifiable if we can get a hold of information that can be linked to the person, such as an IP address or DNS requests. For each subject of an anonymity set, the attacker assigns a probability p_i . The entropy of the system after the attack, $H(X)$, is compared to the maximum entropy, H_M . The degree of anonymity provided by the system is:

$$d = \frac{H(X)}{H_M} = \frac{-\sum_{i=1}^N p_i \log_2(p_i)}{\log_2(N)}$$

For the case $d = 0$, a subject appears as the source of an action with probability 1. When $d = 1$, all subjects equiprobably appear as the source of an action.

Assuming the adversary has no prior knowledge with regard to the anonymity set, for the best case the adversary has no better than $\frac{1}{N}$ chance of identifying a subject within the anonymity set of size N . Therefore, we obtain that all subjects has equiprobable probability to appear as the source of an action.

Anonymity on-demand. In case of incremental path building, keeping the information of the complete path in one node and consequently in the complete message structure may compromise the anonymity of the sender. In loose routing, the complete route is unknown and can be dynamically changed or extended by a router. Consequently, anonymity is increased when the route structure is dynamic and can be altered by any router. Incremental path building is easier to implement and requires less coordination among different routers for the reason that messages are set to be of constant size.

The estimated probability in Section 5 for incremental path building would be the same for the probability of at least one node along the overall path to be a non-compromised node, and the probability in Section 5 in case of loose routing. According to estimated calculations it is highly unlikely for the whole route to be completely compromised. There is a 99% chance for at least one node to be an honest node. Moreover, these calculations show that we can have more than half of compromised nodes and yet remain anonymous in the network.

Using the same traffic analysis as in Section 6 we can assert the anonymity of the system. We obtained the result that the probability for every path of a message is equally likely, and consequently, the probability for each hidden state of the system would be equally likely either. The computation of anonymity is based on entropy computation of the distribution [42, 13]. Since all hidden states are equiprobable, the distribution of the hidden states would be uniform

and the entropy of the system would be maximized.

Similarly to discussion in Section 6, without any additional knowledge of the system, given that the probability for each hidden state of the system is equally likely, and due to symmetric properties of expander graphs each of the subjects in the anonymity set is assigned the probability of $p_i = \frac{1}{N}$ being the sender/receiver of the message. Therefore, the entropy of the system would be the possible maximum.

Attacks mitigation. We now move on to show how certain common attacks on anonymity networks, in particular onion routing based, can be mitigated by the use of our network construction. A survey of attacks on onion routing networks can be found in [21].

Intersection attacks are used to narrow down the possibilities of suspects among a monitored user set by looking up the set of active users in the system. Intersection of these groups allows identification of communicating participants. Mitigation of these attack can be achieved by using cover traffic, upon each output message the node sends a packet on all of its incident edges. Since the constructed overlay network is of constant degree, only a linear number of packets should be sent as a dummy traffic. There, in the worst case, only $O(d_{max}N)$ messages are required, where N is the number of nodes and d_{max} is the maximum degree in graph.

Predecessor timing attack is described in [47], where it is assumed that the set of relays cooperate with each other. Using consistency in timing, attackers can identify whether they are on the same path. Particularly, whether one of the attackers follows the sender on the path.

Unknown path length can significantly decrease the success of the attack [47]. As proposed in our message sending scheme, varying number of intermediate nodes and possibly in combination of loose routing can be used for dynamic path length, which can decrease the success of the attack, or at least impair the confirmation of the attack's success. The amount of rounds required to perform the attack with high probability is $O((\frac{n}{c})^2 \ln n)$ where c is the number of attackers [47]. The required amount of rounds is greater than the route length, which is of order of $O(\log n)$. This means that until the attack is successfully completed, the anonymity network service can be shut down since messages have already arrived to their destinations.

Circuit clogging, or congestion attack, reveals a connection path by overloading certain relay and observing the affected circuits [21]. This attack is easily prevented with the policy we proposed for the constructed network. The message sending scheme, using Valiant randomization technique, balance the load on the relays of the network. It would be extremely difficult to congest a message path, if possible at all, due to the randomization policy of the network. Additionally, loose routing can impair the success of the attack by dynamically modifying a path.

Similar reasoning tells us that resource attacks and DoS attacks on relays, in which an attacker keeps certain relays busy in order to influence circuit construction [21], would be extremely difficult to realize, especially, if loose routing is used. This sort of attack can be looked at if certain nodes are compromised, as such, we have seen in Section 5 that more than half of nodes in the network can be compromised without significantly affecting the routing of traffic.

8 Discussion

We have presented in this paper methods for constructing a flexible, on-demand network service over SDN enabled architecture with defined policy of message sending scheme which can be deployed by service providers, commercial companies, or private users. We have shown that constructing overlay network with expansion properties in combination of randomized message sending policy, we were able to achieve uniform dispersion of network traffic and consequently with high distribution monitoring the network requiring relatively small number of monitors, in particular $O(n/\log n)$ monitors are enough to cover the complete overlay network when each path is of length $O(\log^2 n)$ physical edges. The overlay network can be constructed in distributed

manner, converging faster towards the required features. The constructed overlay network graph is a sparse connected graph, approximating the primal weighted graph if $O(n \log n / \epsilon^2)$ edges are included in the construction.

Further, we have shown that our construction method can be applied for providing anonymity network. This work presents, first of its kind as far as the authors know, anonymity on-demand network service. The NaaS architecture of the network can mitigate most of the known attacks to anonymity networks. Notably, attacks that congest the network or result in relay server denial of service can be coped with on-demand nature of the network. If the user, who deployed the anonymity network, suspects that overtime the relays are compromised, he/she can periodically shut down the service and re-deploy it with different nodes and even with different topology. This kind of service can be suitable for private communication use among trusting parties.

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